Naam:
Adres:
Postcode en Woonplaats:

Geb. datum:
Studierichting:
Jaar van eerste inschrijving:

Bladnr.: 1
Tentamen:
Datum: 14-3-'96
Naam docent:

g 3

1. werk met het bekende isomorfisme:

$D \leftrightarrow s^2+s-2$ inverse: $\frac{1}{s^2+s-2} = \frac{\frac13}{s-1} + \frac{-\frac13}{s+2}$

$\leftrightarrow \frac13 Y(e^x - e^{-2x})$

dus $E := (D\delta)^{-1} = \frac13 Y(e^x - e^{-2x})$ ✓

2. $DT = Y \Rightarrow E * D\delta * T = E * Y \Rightarrow T = E * Y$.

$\leftrightarrow \frac{\frac13}{(s-1)s} + \frac{-\frac13}{(s+2)s} = \frac13\left(\frac{1}{s-1} + \frac{-1}{s}\right) - \frac13\left(\frac{-\frac12}{s+2} + \frac{\frac12}{s}\right)$

$= \frac{\frac13}{s-1} - \frac{\frac12}{s} + \frac{\frac16}{s+2} \leftrightarrow Y\left(\frac13 e^x - \frac12 + \frac16 e^{-2x}\right)$ ✓

3. $(Yf)' = Yf' + \delta$ (sprongregel: $f(0)=1$)

$(Yf)'' = Yf'' + \delta' + \delta$ $\quad f'(0)=1$

$DYf = YDf + 2\delta + \delta' = Yg + 2\delta + \delta'$

$\Rightarrow Yf = E*(Yg + 2\delta + \delta') = Y(\frac13 e^x - \frac13 e^{-2x}) * (Yg + 2\delta + \delta')$

in klassieke vorm:

$f(x) = \int_0^x g(x-y)\left(\frac13 e^y - \frac13 e^{-2y}\right)dy + \frac23 e^x - \frac23 e^{-2x} + \frac13 e^x + \frac23 e^{-2x}$

$= \frac13 \int_0^x g(x-y)(e^y - e^{-2y})\,dy + e^x, \quad x \geq 0$ ✓

voor $x<0$ klopt het ook (vul maar in)

4. $Yf = E * (Y + 2\delta + \delta') = Y\left(\frac13 e^x - \frac12 + \frac16 e^{-2x}\right) + Ye^x$

$\Rightarrow f(x) = \frac43 e^x - \frac12 + \frac16 e^{-2x}$ ✓

II. 1 bekend: $\mathcal{F}(e^{-\pi x^2}) = e^{-\pi y^2}$

vervang nu x door $\frac{x}{\sqrt{2\pi t}}$: $\quad \mathcal{F}(f(\lambda x)) = \frac{1}{\lambda}(\mathcal{F}f)\left(\frac{x}{\lambda}\right)$

$\mathcal{F}\left(\frac{1}{\sqrt{2\pi t}} e^{-x^2/2t}\right) = \frac{1}{\sqrt{2\pi t}} \mathcal{F}\left(e^{-\pi\left(\frac{x}{\sqrt{2\pi t}}\right)^2}\right)$

1

$= \frac{\sqrt{2\pi t}}{\sqrt{2\pi t}}\left(e^{-\pi(\sqrt{2\pi t}\,y)^2}\right) = e^{-\pi^2 2t y^2}$ ✓

2. $\mathcal{F}(G_t * G_s) = \mathcal{F}(G_t)\mathcal{F}(G_s) = e^{-\pi^2 2(t+s)y^2} = \mathcal{F}(G_{t+s})$

$\mathcal{F}$ is injectief dus

$G_t * G_s = G_{t+s} \quad \forall t,s>0.$ ✓

3. Zij T een getemperde distributie en $\varphi \in S$ dan geldt $\langle \mathcal{F}(T), \varphi \rangle = \langle T, \mathcal{F}(\varphi) \rangle$ ✓

Fout ! en zonder betekenis

We hebben nodig: $\mathcal{F}(1) = \delta$ (bekend)

$\mathcal{F}(\lambda f) = \lambda \mathcal{F}(f)$ $\quad$ $\mathcal{F}(f') = 2\pi i x \, \mathcal{F}(f)$

dus $\mathcal{F}(x^2) = \dfrac{\mathcal{F}(2x)}{2\pi i x} = \dfrac{\mathcal{F}(2)}{(2\pi i x)^2} = \dfrac{2\mathcal{F}(1)}{-4\pi^2 x^2} = \dfrac{\delta}{-2\pi^2 x^2}$

$\mathcal{F}(f'') = (2\pi i x)^2 \mathcal{F}(f) = -4\pi^2 x^2 \mathcal{F}(f)$

III 1. $T_h = \dfrac{\delta_h - \delta_{-h}}{2h}$ $\quad$ $\mathcal{F}(\delta'') = -4\pi^2 x^2$ (even) $\quad$ $\mathcal{F}(x^2) = \dfrac{-\delta''}{4\pi^2}$

$\langle T_h, \varphi \rangle = \frac{1}{2h}(\langle \delta_h, \varphi \rangle - \langle \delta_{-h}, \varphi \rangle) = \dfrac{\varphi(h) - \varphi(-h)}{2h} \xrightarrow[h \to 0]{\text{L'Hôp.}} \dfrac{\varphi'(0) + \varphi'(0)}{2}$

$= \varphi'(0) = \langle -\delta', \varphi \rangle$

dus $\lim_{h \to 0} T_h = -\delta'$. ✓

2. $\lim_{h \to 0} T_h * T_h = \lim_{h \to 0} T_h * \lim_{h \to 0} T_h = -\delta' * -\delta' = \delta' * \delta' = \delta''$.

$T_h * T_h = \dfrac{\delta_h * \delta_h - 2\delta_h * \delta_{-h} + \delta_{-h} * \delta_{-h}}{4h^2} = \dfrac{\delta_{2h} - 2\delta_0 + \delta_{-2h}}{4h^2}$

dus $\varphi''(0) = \langle \delta'', \varphi \rangle = \langle \lim_{h \to 0} T_h * T_h, \varphi \rangle = \lim_{h \to 0} \langle \dfrac{\delta_{2h} - 2\delta_0 + \delta_{-2h}}{4h^2}, \varphi \rangle$

$= \lim_{h \to 0} \dfrac{\varphi(2h) - 2\varphi(0) + \varphi(-2h)}{4h^2}$ ✓

3. $\left(\dfrac{\delta_h - \delta_{-h}}{2h}\right)^{*n} = \dfrac{1}{(2h)^n} \sum_{k=0}^{n} (\delta_h)^{*k} * (\delta_{-h})^{*(n-k)} \binom{n}{k}$

$= \dfrac{1}{(2h)^n} \sum_{k=0}^{n} (-1)^{n-k} \delta_{hk - h(n-k)} \binom{n}{k}$

$= \dfrac{(-1)^n}{(2h)^n} \sum_{k=0}^{n} (-1)^k \delta_{h(n+2k)} \binom{n}{k}$

4. $\lim_{h \to 0} (T_h)^{*n} \underset{\text{cont.}}{=} (\lim_{h \to 0} T_h)^{*n} = (-\delta')^{*n} = (-1)^n \delta^{(n)}$

$\varphi^{(n)}(0) = \langle (-1)^n \delta^{(n)}, \varphi \rangle = \langle \lim_{h \to 0} (T_h)^{*n}, \varphi \rangle =$

$= \lim_{h \to 0} \langle \dfrac{1}{(2h)^n} \sum_{k=0}^{n} (-1)^{n-k} \delta_{hk - h(n-k)} \binom{n}{k}, \varphi \rangle$

2

$= \lim_{h \to 0} \dfrac{1}{(2h)^n} \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} \varphi((2k-n)h)$

$= \lim_{h \to 0} \dfrac{\varphi(nh) - \binom{n}{1}\varphi((n-2)h) + \binom{n}{2}\varphi((n-4)h) - \ldots \cancel{\pm \varphi(-nh)}}{(2h)^n}$

Afdeling Wiskunde en Informatica R.U.G.

Naam:	Geb. datum:	Bladnr.: 2.
Adres:	Studierichting:	Tentamen:
Postcode en Woonplaats:	Jaar van eerste inschrijving:	Datum: 14-3-'96
		Naam docent:

III. 5. $\langle T_n * S_n - T*S, \varphi \rangle =$

$\langle T_n * S_n, \varphi\rangle - \langle T_n * S, \varphi \rangle + \langle T_n * S, \varphi\rangle - \langle T*S, \varphi\rangle =$

$\{ \Phi(x,y) = \varphi(x+y)\ ,\ \Phi_x : y \mapsto \varphi(x+y) \}$.

$\langle T_n, \langle S_n, \Phi_x\rangle\rangle - \langle T_n, \langle S, \Phi_x\rangle\rangle + \langle T_n, \langle S, \Phi_x\rangle\rangle - \langle T, \langle S, \Phi_x\rangle\rangle$

$= \langle T_n, \langle S_n - S, \Phi_x\rangle\rangle + \langle T_n - T, \langle S, \Phi_x\rangle\rangle$?

$\to \langle T_n, \langle 0, \Phi_x\rangle\rangle + \langle 0, \langle S, \Phi_x\rangle\rangle = 0$

3